Indian Statistical Institute
Second Semester Back Paper Exam 2003-2004
M.Math I Year
Functional Analysis

Time: 3 hrs Date:20-07-04 Max. Marks : 100

1. Let X be a n-dimensional normed linear space. Let $L : X \to C^n$ be a linear map. Show that L is continuous. [15]

2. Let $M = \{f \in C([0,1]) : f'$ exists and is continuous$\}$. Define $||f||* = ||f|| + ||f'||$. Show that $||||*$ is a norm on M. [10]

3. State and prove the open mapping theorem. [15]

4. Let X and Y be Banach spaces. Let $T \in L(X,Y)$ be a compact operator. Show that T^* is a compact operator. [15]

5. Let $\{f_n\}_{n\geq 1} \subset L^2(R)$ be a complete ortho normal sequence. Define $\Psi : L^2(R) \to \ell^2$ by $\Psi(f) = (\int f\bar{f}_n dx)_{n\geq 1}$. Show that Ψ is an onto isometry. [15]

6. Let H be a Hilbert space. Suppose $N \in L(H)$ is a normal operator. Show that λ is an eigen value of N if and only if $\bar{\lambda}$ is an eigen value of N^*. [15]

7. Let K be a compact Hausdorff space. Let $f : K \to \ell^2$ be a continuous map. Define $T : \ell^2 \to C(K)$ by $T(\alpha)(k) =< \alpha, f(k) >$. Show that T is a well-defined, bounded linear map. [15]